Year 12 Methods Units 3,4
Test 1 2020

Section 1 Calculator Free

Differentiation, Applications of Differentiation, Integration, Applications of Integration

STUDENT'S NAME ______________________________

DATE: Friday 6th March **TIME:** 20 minutes **MARKS**: 18

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

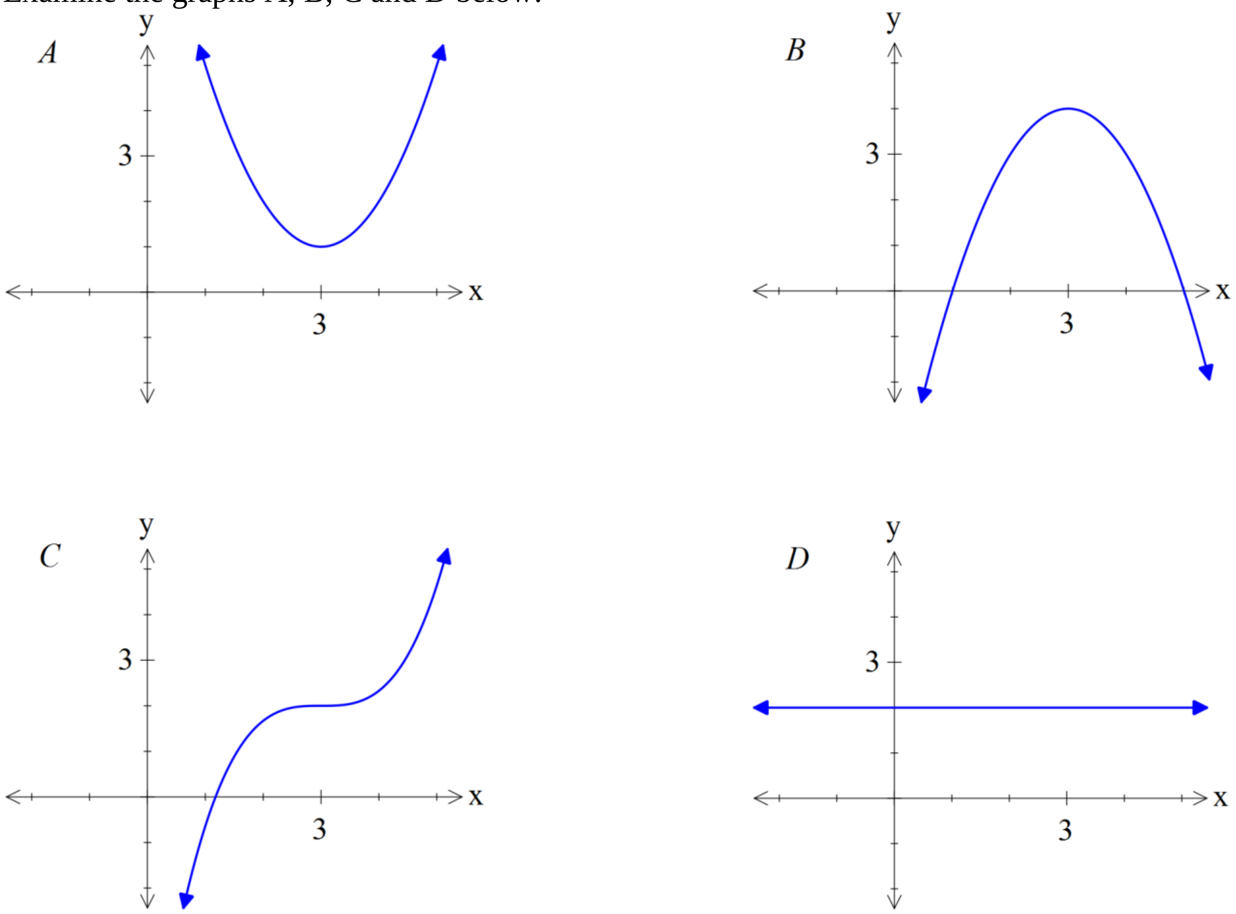
Determine each of the following

(a) $\int \frac{3x-5}{x^4} dx$ [2]

(b) $\int \; 6(2x+3)^5 dx$ [3]

2. (4 marks)

Examine the graphs A, B, C and D below:



List ALL graphs in which

(a) $f'(3) = 0$

(b) $f'(4) > 0$

(c) $f''(3) < 0$

(d) $f''(4) > 0$

3. (4 marks)

Determine the area bound by the x-axis and $y = 6 + 5x + x^2$.

4. (5 marks)

Determine the equation of the curve with a minimum turning point of $(3, -5)$ and a second derivative of $12x - 18$.

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Year 12 Methods Units 3,4
Test 1 2020

Section 2 Calculator Assumed

Differentiation, Applications of Differentiation, Integration, Applications of Integration

STUDENT'S NAME ____

DATE: Friday 6th March **TIME:** 30 minutes **MARKS**: 33

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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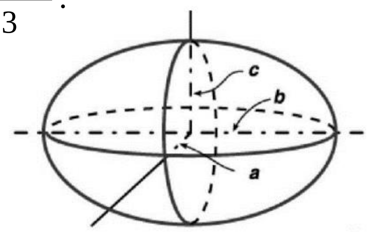
5. (4 marks)

A general ellipsoid has semi-axes lengths a, b and c as shown in the diagram below and has a volume given by $V = \frac{4\pi abc}{3}$.



Consider the ellipsoid where the relationship between the semi-axes lengths is that b is three times a , and that the sum of a and c is 42 cm.

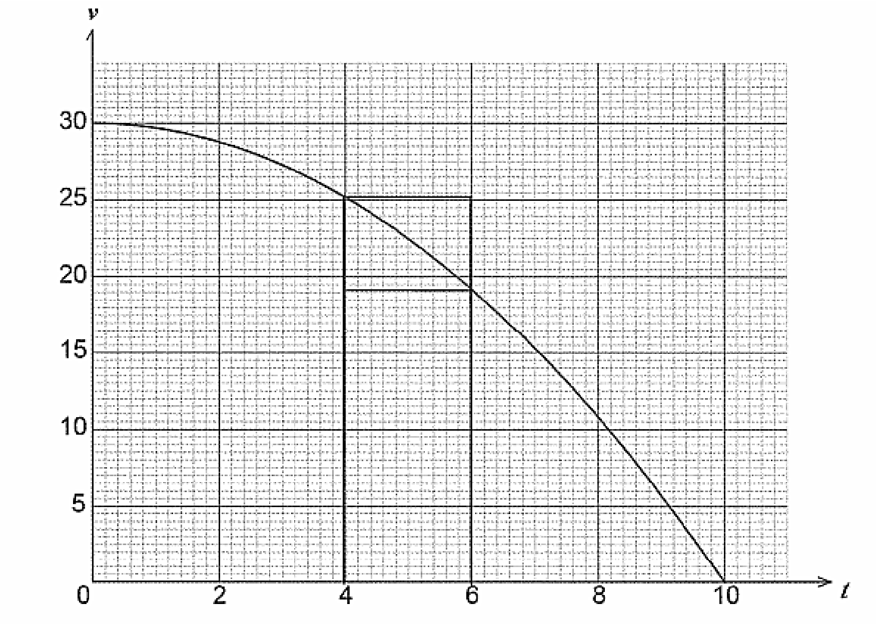
(a) Show that the volume of this ellipsoid is given by $168\pi a^2 - 4\pi a^3$. [1]

(b) Use the increments formula to estimate the change in volume of the ellipsoid when a increases from 30 cm to 30.5 cm. [3]

6. (6 marks)

A train is travelling at 30 metres per second when the breaks are applied. The velocity of the train is given by the equation $v = 30 - 0.3t^2$ where t represents the time in seconds after the breaks are applied.

The velocity- time graph is shown below.



The area under the velocity-time graph gives the total distance travelled for a particular time period.

(a) Complete the tables below and estimate the distance travelled by the train during the first 6 seconds by calculating the mean of the rectangles (overestimate and underestimate). The rectangles for the 4-6 seconds interval are shown in the table below.

Time (t)	0	2	4	6
Velocity (v)		28.8		19.2

Rectangle	0-2	2-4	4-6	Total
Underestimate			38.4	
Overestimate			50.4	

Estimated distance travelled: ____________________ [5]

(b) The exact distance travelled is during the first 6 seconds is 158.4m.
How could you determine a better estimate of the distance travelled by the train during the first six seconds than the one determine in (a)?

7. (10 marks)

The displacement of a particle moving along a straight line at time t seconds is given by $s = t^3 - 4t^2 + 4t - 10$ metres.

(a) Determine the change in displacement in the first 2 seconds. [2]

(b) Determine the velocity of the particle when $t = 5$ seconds. [2]

(c) Determine when the particle is instantaneously at rest. [2]

(d) Determine the initial acceleration of the particle. [2]

(e) Determine the distance travelled in the first 2 seconds. [2]

8. (6 marks)

A company's revenue each week is $\$(800 + 1000n - 20n^2)$ where n is the number of employees. The company spend \$760 per employee for wages and materials.

(a) Write an expression for the company's weekly profit, P dollars. [1]

(b) Determine the number of employees required for maximum profit and hence calculate the maximum weekly profit. [3]

(c) Determine the marginal revenue when $n = 4$ and explain what this tells us about revenue. [2]

9. (7 marks)

A man at point A on the bank of a river 2km wide wishes to reach a point B, 4km down from point A on the opposite bank. He can travel in a boat at 6km/hr and ride a bicycle at 10km/hr.

(a) Given that Time = $\frac{\text{Distance}}{\text{Speed}}$, show that the equation for the time taken for the man to reach the point across the river is $t = \frac{\sqrt{4+x^2}}{6} + \frac{4-x}{10}$ where x is the distance landed downstream by the man on the opposite bank. [3]

(b) Determine how far downstream he must land on the opposite bank in order to reach point B in a minimum time and state that minimum time. [4]